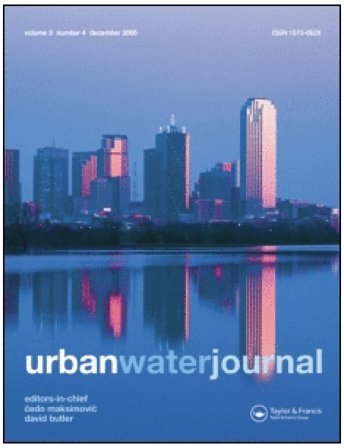

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RESEARCH ARTICLE

Identification of segments and optimal isolation valve system design in water distribution networks

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This paper presents a novel methodology for assessing an isolation valve system and the portions of a water distribution network (segments) directly isolated by valve closure. Planned (e.g. regular maintenance) and unplanned interruptions (e.g. pipe burst) occur regularly in water distribution networks, making it necessary to isolate pipes. To isolate a pipe in the network, it is necessary to close a subset of valves which directly separate a small portion of the network, i.e., causing minimum possible disruption. This is not always straightforward to achieve as the valve system is not normally designed to isolate each pipe separately (i.e. having two valves at the end of each pipe). Therefore, for management purposes, it is important to identify the association between each subset of valves and the segments directly isolated by closing them. Furthermore, it is also important to improve the design of the isolation valve system in order to increase network reliability. Thus, this paper describes an algorithm for identifying the association between valves and isolated segments. The approach is based on the use of topological matrices of a network whose topology is modified in order to account for the existence of the valve system. The algorithm is demonstrated on a simple network and tested on an Apulian network where the isolation valve system is designed using a classical multi-objective optimisation using genetic algorithms.

Keywords: water distribution systems; network analysis; topological analysis; reliability analysis; isolation valves

Introduction

Analysis of the isolation valve system should always be performed as part of the assessment of system reliability (Walski 1993a,b, 2002). The aim of an isolation valve system is to separate a portion of the water distribution network for management purposes such as, for example, planned and unplanned rehabilitation actions. Furthermore, concern over potential acts of sabotage against water distribution systems has escalated in recent years, putting pressure on system operators to implement policies and strategies aimed at protecting water supply infrastructure in real time. A number of studies have addressed the subject, focusing on the development of methodologies for the optimal positioning of monitoring stations against intentional interference (Ostfeld and Salomon 2004, Berry *et al*. 2005, Cozzolino *et al*. 2005), and the design of early warning contaminant detection systems (Ostfeld *et al*. 2004, Guan *et al*. 2006). However, once the warning system is developed and in place, further analyses are needed to limit the extent of damage by, for example, trying to isolate the incident by closing off a portion of the system.

Thus, a new challenge is to design an isolation valve system that, coupled with a contaminant-detection system, can act as part of real-time control to diffuse threats in the network (Davidson *et al*. 2005). This can be achieved by better understanding and utilisation of isolation valves. For example, this approach could be used to isolate contaminated portions of the network.

Jun and Loganathan (2007) have recently proposed a method for the identification of network segments, which are portions of the network that can be isolated by closing valves (Walski 1993a,b, 2002), and unintended isolation of the network. The unintended isolation is a portion of the network disconnected from water source(s) as the secondary effect of segment isolation. Kao and Li (2007) have also proposed the automatic identification of segments and 'unintended segments' in order to optimise pipe replacement based on an existing isolation valve system.

This paper presents a new method for identifying the isolated segments, both intended and unintended, based on topological incidence matrices commonly used in network hydraulic models and on the

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modification of the original network topology. The list structures (Cormen *et al*. 2001, Walski *et al*. 2006, Jun and Loganathan 2007) could also be used to perform the same task. However, the approach developed here has the potential advantage of being more easily understood by users without a specific background/training in graph theory, as it uses the basic matrix structure commonly employed in network simulation models.

The algorithm has been applied to an example network in order to demonstrate its performance on a simple case study. It has then been applied to a real network in order to optimise the isolation valve system for system reliability purposes. The design of an isolation valve system was chosen as for the new methodology. The feasibility and robustness of the methodology, which involves genetic algorithm (Goldberg 1989) optimisation with its stochastic exploration of candidate solutions, is thoroughly tested as a large number of isolation valve system scenarios are analysed during an optimisation run. However, the proposed algorithm is intended not only for planning of new networks, but also when existing hydraulic systems and reliability of their isolation valve systems are analysed. For example, this is important when engineers consider improvements to the existing isolation valve system or when network upgrade or extension is considered.

$$\bar{\mathbf{A}}_{pn}(k,j) = \begin{cases} -1 & \text{if assumed flow direction is from node } j \text{ to node } i \\ 0 & \text{if node } i \text{ is not connected to node } j \\ 1 & \text{if assumed flow direction is from node } i \text{ to node } j \end{cases} \quad (1)$$

Rules and network topology for analysing valve system

The network in Figure 1, comprising, n_n (=6) internal nodes with unknown pressure heads and n_o (=1) nodes with known heads (the tank) and n_p (=8) pipes, will be used to illustrate the approach.

The matrix formulation used for network simulation models (Todini and Pilati 1988, Todini 2003, Giustolisi *et al*. 2008a) will be used here in order to define the network topology. Therefore, $\mathbf{A}_{pn} = \mathbf{A}_{np}^T$ and $\mathbf{A}_{p0}$ are topological incidence sub-matrices, of size (n_p,n_n) and (n_p,n_0) respectively, derived from the general topological matrix $\bar{\mathbf{A}}_{pn}$ of size [n_p, $n_n + n_0$], the definition of which is as follows:

Thus, the matrix $\bar{\mathbf{A}}_{pn}$ of the network in Figure 1 is reported in Table 1. The columns N*i* correspond to *i*-th node (of known and unknown heads, considering the hydraulic simulation perspective) and the rows P*k* identify the *k*-th pipe in network topological matrix. In what follows the *k*-th pipe and the *i*-th node will be optionally identified with the previous labels.

Now, let us assume the isolation valve system configuration of Figure 1. The issue is the identification

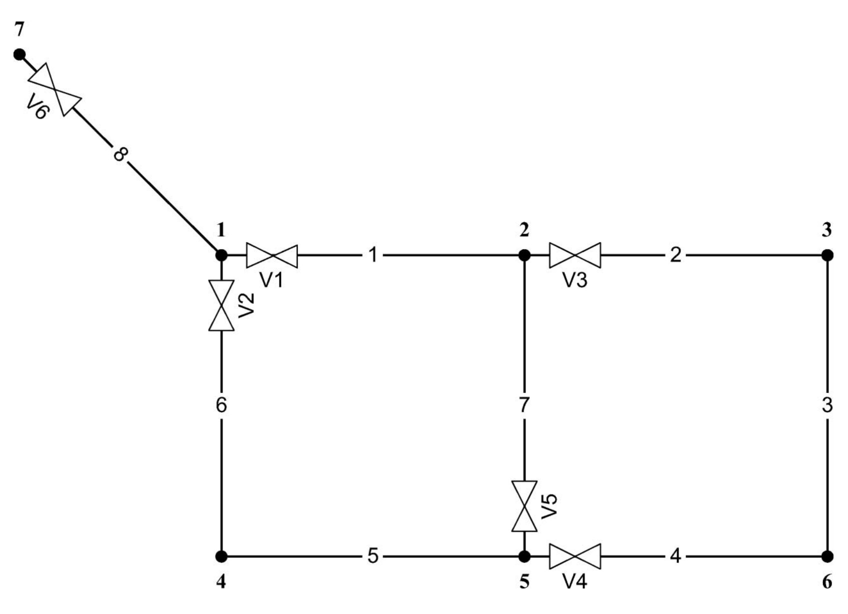


Figure 1. Example network topology and isolation valve system scenario.

of each segment associated with each subset of the isolation valve system. A segment is defined here as a portion of the network constituted by one or more pipes and nodes which remain isolated (Walski 1993a). Thus, a subset of isolation valves is a group of valves which must be closed in order to separate a segment. Clearly, the association between a subset of isolation valves and a segment is unique, but the number of solutions increases combinatorially if no additional constraint is considered.

This constraint is the general technical requirement to isolate a minimum number of pipes and nodes using a set of isolation valves. This set of pipes is a minimal cut set, or a set that contains the fewest pipes, such that all pipes in the set have become disconnected. Thus, each subset of valves will be characterised by: (1) a minimal cut set of pipes and nodes (segment), which can be isolated in the network; and (2) a unique two-way association between them. Then, a revised definition of segment is 'the smallest portion of a distribution system that can be isolated by a subset of valves'. For example, the 'N valve rule' scenario (two valves located at each pipe end or one valve at each pipe connected to the node) identifies the association between each segment, constituted by one pipe, and the two valves located on that pipe. The number of segments is equal to the number of pipes and the number of valves is twice the number of the segments/pipes. For the isolation valve system in Figure 1, the automatic identification of the two-way associations between valves and segments is less trivial than in the 'N valve rule' and becomes increasingly difficult with increasing network size.

In order to identify a network isolation valve system, let us define the matrix $\mathbf{V}_{pn}$ whose elements $v(k,i)$ are associated to the valve on k-th pipe and the nearest (i-th) node. Thus, the valves in the system of Figure 1 are identified by matrix elements $v(1,1)$, $v(6,1)$, $v(2,2)$, $v(4,5)$, $v(7,5)$ and $v(8,7)$, labelled V1, V2, V3, V4, V5 and V6, respectively. The valve labels are assigned using the ordering rule whereby sorting is performed firstly based on node index i and then based on pipe index k. Thus, the general topological matrix of the isolation valve system is reported in Table 2.

Obviously, the matrix $\mathbf{V}_{pn}$ has the same dimension of $\bar{\mathbf{A}}_{pn}$ and its non-zero elements are $v(k,i)$ of the isolation valve system. Furthermore, the signs of $v(k,i)$ are defined using the same convention as for the general topological matrix (see Equation (1)). For example, in the case of the 'N valve rule' scenario the two matrices coincide.

Once the general topological matrix of the isolation valve system is defined, the next step is to create a modified network topology. This new topology will allow identification of the associations between each subset of valves and a segment (the smallest portion of network composed of one or more pipes and nodes that can be isolated). The network topology depicted in Figure 2 is helpful for the association of valve-segments. The new topology is derived from that in Figure 1 by adding one pseudo-pipe (numbered from 9 to 14 in Figure 2) for each valve. This modified topology permits the analysis of the association of valve-segments.

The usefulness of pseudo-pipes in the analysis is shown in Figure 3, where all the valves are closed and gaps are generated where the associated pseudo-pipes have been removed. By considering that each valve is connected to the network at both of its ends (Figure 3), one valve end connected to the k-th pipe and the other to the i-th node, each subset associated to a segment is composed of valves which are connected to that segment by one side (pipe k or node i). Furthermore, each segment is composed of pipes and nodes connecting the associated subset of valves. Clearly, each isolation valve can belong to a maximum of two valve subsets associated with two segments. However, a valve may not belong to any subset, in which case it is not necessarily an isolation valve. For example, in Figure 4 the valves V1 and V2 (Figure 3) are removed to illustrate this point. Consequently, the valve V3 does not belong to any subset of valves because the path of pipes 7–1–6–5 connect its two sides.

Using the modified network in Figure 3, Table 3 shows the relationship between each valve subset and a network segment from Figure 1. The gaps indicate

Table 1. General topological matrix of the example network in Figure 1.

	N1	N2	N3	N4	N5	N6	N7	
$\bar{\mathbf{A}}_{pn} = [\mathbf{A}_{pn} \vert \mathbf{A}_{p0}] =$	−1	1	0	0	0	0	0	P1
	0	−1	1	0	0	0	0	P2
	0	0	−1	0	0	1	0	P3
	0	0	0	0	1	−1	0	P4
	0	0	0	1	−1	0	0	P5
	−1	0	0	1	0	0	0	P6
	0	−1	0	0	1	0	0	P7
	1	0	0	0	0	0	−1	P8

Table 2. General topological matrix of the isolation valve system of the network in Figure 1.

	N1	N2	N3	N4	N5	N6	N7	
$\mathbf{V}_{pn} =$	−1	0	0	0	0	0	0	P1
	0	−1	0	0	0	0	0	P2
	0	0	0	0	0	0	0	P3
	0	0	0	0	1	0	0	P4
	0	0	0	0	0	0	0	P5
	−1	0	0	0	0	0	0	P6
	0	0	0	0	1	0	0	P7
	0	0	0	0	0	0	−1	P8

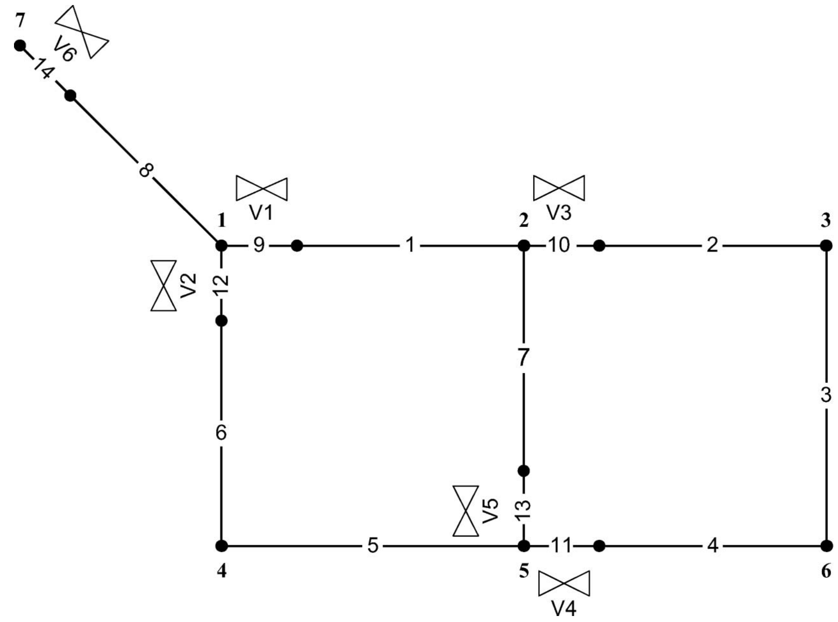


Figure 2. Modified network topology ($[\mathbf{B}_{pn}\ \mathbf{V}_{pv};\ \mathbf{V}_{vn}\ \mathbf{V}_{vv}]$ matrix).

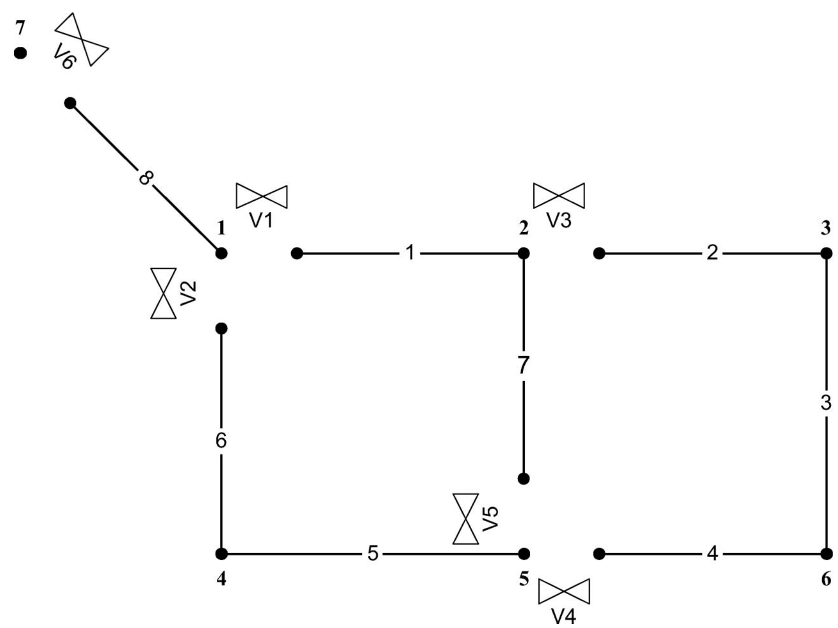


Figure 3. Modified network topology with gaps ($[\mathbf{B}_{pn}\ \mathbf{V}_{pv}]$ matrix).

valves assumed closed and follow the rule set out above.

It is worth noting that the valve V6 in the network of Figure 1 is required because the node 7 is the source node and pipe 8 cannot be isolated without that valve. Therefore, some valves may be useful in order to isolate a single node. Thus, generalising the definition of the segment concept as the portion of the network which could contain just a single node, some valve subsets could be associated with single-node segments. This type of segments, i.e., that correspond to nodes with one valve for each pipe joining at the junction, are composed of a short pipe section between each valve and the junction. For example, these special segments allow consideration of the source disconnection in the analysis of the isolation valve system. Thus, each valve belongs to two subsets and each pipe and node of the network belongs to one segment. Therefore, the last

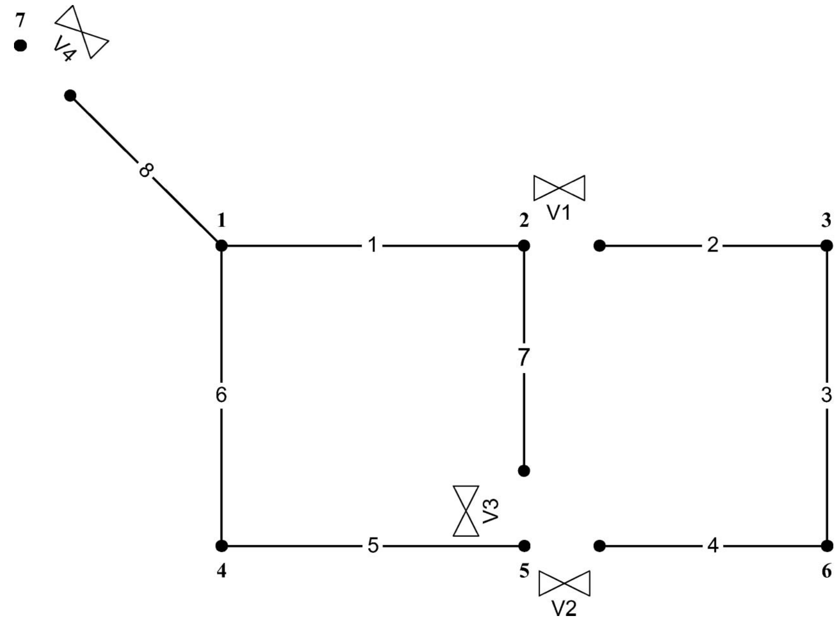


Figure 4. Isolation valve system scenario generating a non-useful valve (V3).

Table 3. Association between subset of valves and segments (pipes and nodes) of the network in Figure 1.

Segments	Valve labels
P8 and N1	V1-V2-V6
P1-P7 and N2	V1-V3-V5
P2-P3-P4 and N3-N6	V3-V4
P5-P6 and N4-N5	V2-V4-V5
N7	V6

row in Table 3 reports the association between a subset of valves (one valve in this case) and the special segment composed of one node (the tank node in this case).

It is worth noting that, although the valve analysis has been performed considering a maximum of two valves for each pipe close to its ending nodes, it is possible to add dummy nodes when considering additional valves along a pipe.

Finally, the trivial case of the 'N valve rule' scenario is now revised considering that each valve $v(k,i)$ is useful in order to isolate: (i) the k-th pipe together with the other valve on the same pipe and; (ii) the i-th node together with the other valves close to that node. Therefore, the number of segments is equal to the number of pipes plus the number of nodes (of known and unknown heads).

Automation of the network topology modification

Now, the next issue is to generate automatically the topology of the network in Figure 2 in order to analyse the valve system in Figure 1. The general topological matrix of the modified network topology can be achieved using the following matrix composed of four building blocks

$$\bar{\mathbf{A}}_{pn-mod} = \begin{bmatrix} \mathbf{B}_{pn} = \bar{\mathbf{A}}_{pn} - \mathbf{V}_{pn} & \mathbf{V}_{pv} \\ \mathbf{V}_{vn} & \mathbf{V}_{vv} \end{bmatrix} \tag{2}$$

where the matrix $\mathbf{B}_{pn}$ is generated from $\bar{\mathbf{A}}_{pn}$, the original general topological matrix of the system, substituting the value zero in the positions of elements associated with the valves (i.e. subtracting $\mathbf{V}_{pn}$). The other three matrices are generated in the following ways:

- $\mathbf{V}_{pv}$ is obtained by removing the columns of zeros in $\mathbf{V}_{pn}$ and splitting the remaining columns that have more than one non-zero element into multiple columns in order to obtain one element in each column.
- $\mathbf{V}_{vn}$ is obtained in the same way as $\mathbf{V}_{pv}$ but acting on the transpose of $\mathbf{V}_{pn}$.
- $\mathbf{V}_{vv}$ is obtained in the same way as $\mathbf{V}_{pv}$ but acting on the transpose of $-\mathbf{V}_{vn}$.

Thus, the matrix $\mathbf{V}_{pv}$ has the number of rows equal to the original number of pipes (real pipes) and the number of columns equal to the number of valves (the column number matches the label number because of the rule used for generating the matrix itself). This building block, together with that of $\mathbf{B}_{pn}$, is useful for duplicating the i-th node associated with the valve $v(k,i)$ along the k-th pipe. For example, the network reported in Figure 3 is characterised by the building blocks $\mathbf{B}_{pn}$ and $\mathbf{V}_{pv}$ of the matrix $\bar{\mathbf{A}}_{pn\text{-}mod}$.

The matrix $\mathbf{V}_{vn}$ has the number of columns equal to the original number of nodes (internal nodes and known heads) and the number of rows equal to the number of valves. Finally, the matrix $\mathbf{V}_{vv}$ has the number columns and rows equal to the number of valves in the system.

These last two building blocks of the matrix $\bar{\mathbf{A}}_{pn\text{-}mod}$ are useful for generating the 'pseudo' pipes corresponding to valves in the system. For example, the system matrix $\bar{\mathbf{A}}_{pn\text{-}mod}$ for network in Figure 2 is shown in Table 4.

Identification of isolation valve-segments association

Once the matrix $\bar{\mathbf{A}}_{pn\text{-}mod}$ is created the next issue is to automatically identify the association between the isolation valve subsets and the segments as reported in Table 1. To achieve this, it is possible to use a method such as the breadth or depth first search algorithm, as adopted in some software packages (Walski *et al.* 2006). A different approach has been used here based on matrix computations which does not require a specific knowledge of graph theory.

Starting from the incidence matrix $\mathbf{C} = [\mathbf{B}_{pn} \mid \mathbf{V}_{pv}]$ it is possible to perform the matrix product,

$$|\mathbf{C}| \times |\mathbf{C}|^T = \mathbf{V} = \mathbf{L} + 2\mathbf{I} \tag{3}$$

where the symmetric matrix $\mathbf{V}$ is the sum of the line or edge graph matrix $\mathbf{L}$ (Brualdi and Ryser 1991) and of the identity matrix $\mathbf{I}$ multiplied by two. The use of the absolute value function in the above equation is necessary in order to consider the undirected version of $\mathbf{C}$ associated with the underlying graph. The matrix $\mathbf{L}$ represents the adjacency between pipes and $2\mathbf{I}$ is the degree or valence of each pipe that is always equal to two, as the number of adjacent nodes is equal to two (Brualdi and Ryser 1991). Thus, considering the elements $i \neq j$, the non-zero $v(i,j)$ of $\mathbf{V}$ in each column (or row) provide the i-th indices of the pipes which are adjacent to j-th pipe. The same applies to non-zero elements of $\mathbf{L}$.

Now, considering $\mathbf{V}^2$,

$$\mathbf{V}^2 = (\mathbf{L} + 2\mathbf{I})^2 = \mathbf{L}^2 + 4\mathbf{L} + 4\mathbf{I} \tag{4}$$

Equation (4) demonstrates that $\mathbf{V}^2$ depends on the sum of $\mathbf{L}^2$ and $\mathbf{L}$. It is well known (Brualdi and Ryser 1991) that $\mathbf{L}^2$ (similarly to the power two of the adjacency matrix) represents in each element $l(i,j)$, considering $i \neq j$, the number of paths of length two. The path length is defined here as the number of nodes internal to the path between the two pipes (Brualdi and Ryser 1991). Therefore, the non-zero elements of $\mathbf{V}^2$ represent the pipes which are connected by path(s) of length less than or equal to two. Considering the power m of $\mathbf{V}$ and its expansion similarly to Equation (4), it is possible to state that the non-zero elements of $\mathbf{V}^{\mathrm{m}}$ represent the pipes which are connected by path(s) of length less of equal to m.

Therefore, transforming each matrix $\mathbf{V}^{\mathrm{m}}$ in the Boolean form (non-zero elements in $\mathbf{V}^{\mathrm{m}}$ are replaced by 1's) a value of m exists for which the matrix $\mathbf{V}^m$ equals $\mathbf{V}^{m-1}$. Consequently, the matrix $\mathbf{V}^{m-1}$ indicates in each column the pipes connected to each other (segments, i.e., subcomponents of the network). It is worth noting that for totally connected networks the matrix $\mathbf{V}^m$ becomes full (i.e., it will not have zero elements), m depending on the maximum of the minimum path lengths between two pipes of the network. The matrix $\mathbf{V}^m$ is a pipe version of the 'transitive closure of the graph' (Brualdi and Ryser 1991).

Table 5 shows the Boolean square symmetric matrix, indicating those pipes of the network segments in Figure 2 which have non-zero values in some unique columns (e.g. the columns P1, P2, P5 and P8 describe

Table 4. System matrix of the network in Figure 2.

		N1	N2	N3	N4	N5	N6	N7		V1	V2	V3	V4	V5	V6	
$\bar{\mathbf{A}}_{pn\text{-}mod} =$	$\mathbf{B}_{pn} =$	0	1	0	0	0	0	0	$\mathbf{V}_{pv} =$	−1	0	0	0	0	0	P1
		0	0	1	0	0	0	0		0	0	−1	0	0	0	P2
		0	0	−1	0	0	1	0		0	0	0	0	0	0	P3
		0	0	0	0	0	−1	0		0	0	0	1	0	0	P4
		0	0	0	1	−1	0	0		0	0	0	0	0	0	P5
		0	0	0	1	0	0	0		0	−1	0	0	0	0	P6
		0	−1	0	0	0	0	0		0	0	0	0	1	0	P7
		1	0	0	0	0	0	0		0	0	0	0	0	−1	P8
	$\mathbf{V}_{vn} =$	−1	0	0	0	0	0	0	$\mathbf{V}_{vv} =$	1	0	0	0	0	0	P9 V1 = $v(1,1)$
		0	−1	0	0	0	0	0		0	0	1	0	0	0	P10 V3 = $v(2,2)$
		0	0	0	0	1	0	0		0	0	0	−1	0	0	P11 V4 = $v(4,5)$
		−1	0	0	0	0	0	0		0	1	0	0	0	0	P12 V2 = $v(6,1)$
		0	0	0	0	1	0	0		0	0	0	0	−1	0	P13 V5 = $v(7,5)$
		0	0	0	0	0	0	−1		0	0	0	0	0	1	P14 V6 = $v(8,7)$

Table 5. Boolean form of $\mathbf{V}^{m-1} = 2$ for the network topology in Figure 3.

	P1	P2	P3	P4	P5	P6	P7	P8	
	1	0	0	0	0	0	1	0	P1
	0	1	1	1	0	0	0	0	P2
	0	1	1	1	0	0	0	0	P3
$\mathbf{V}^{m-1} =$	0	1	1	1	0	0	0	0	P4
	0	0	0	0	1	1	0	0	P5
	0	0	0	0	1	1	0	0	P6
	1	0	0	0	0	0	1	0	P7
	0	0	0	0	0	0	0	1	P8

the pipes of the four segments reported in Table 3). It is also interesting to note that for the 'N valve rule' scenario **V** is an identity matrix.

The calculations of matrix power by recursive product in Equation (4) are fast for sparse matrices. Thus, computation of Equation (4) becomes slower for large networks with few valves (a large number of pipes in each segment, rendering less sparse $\mathbf{V}^m$). Clearly, this is an atypical situation because, the larger the network size, the greater is the number of segments. Furthermore a constraint on the maximum m can be used. For example, in valve system design, solutions with large segments (i.e. m) are not technically useful and can be discarded in the analysis.

Once the segment pipes from the matrix $\mathbf{V}^{m-1}$ (e.g. {P1,P7}, {P2,P3,P4}, {P5,P6}, {P8} in our example case) are identified, the nodes belonging to those segments can be obtained from the matrix $\mathbf{B}_{pn}$ considering for each i-th row (pipe) the j-th non-zero elements (nodes) (e.g. {P1,P7,N2}, {P2,P3,P4,N3,N6}, {P5,P6,N4,N5}, {P8,N1} are the segments for this example case). Clearly each node can belong to only one segment.

The next issue is to associate segments to each valve subset. $\bar{\mathbf{A}}_{pn\text{-}mod}$ is now used because it generates valve association via the building block $\mathbf{V}_{pv}$ and/or using the building block $\mathbf{V}_{vv}$. In fact, as previously indicated, valves are connected to a segment by one of the two attributes (k-th pipe or i-th node). In the first case, the valve labels can be directly read from the matrix $\mathbf{V}_{pv}$ and, in the latter, from the matrix $\mathbf{V}_{vv}$ passing through $\mathbf{B}_{pn}$ and $\mathbf{V}_{vn}$. For example, the segment {P1,P7,N2} has three valves {V1,V3,V5}. Valves V1 and V5 are connected to pipes 1 and 7. Thus, labels V1 and V5 can be obtained by considering the index of non-zero elements of P1 and P7 rows of $\mathbf{V}_{pv}$. Then, for valve V3, column N2 of matrix $\mathbf{V}_{vn}$ reveals that node 2 is connected to pipe 10 (the non-zero element in row P10) and, finally, the matrix $\mathbf{V}_{vv}$ associates the pseudo-pipe indices with valve labels, thus providing the valve label V3 as non-zero-element of the row associated with the pipe 10.

Finally, each column of zeros in $\mathbf{B}_{pn}$ (N7 in our example case) denotes the valve labels (V6 in our case), using $\mathbf{V}_{vn}$ and $\mathbf{V}_{vv}$ as previously done, which correspond to the particular segment constituted by one node (see the last row of Table 3). It is worth noting that the label of a non-useful valve (as in the case of Figure 4) is obtained twice, directly through $\mathbf{V}_{pn}$ and using $\mathbf{V}_{vn}$ and $\mathbf{V}_{vv}$, because it is simultaneously connected to a pipe and a node (node 5 and pipe 7 in the case of Figure 4).

Segments considering source of water: unintended isolation

Finally, the identification of 'unintended isolation' (Jun and Loganathan 2007) is required. When a subset of valves isolates a segment, other parts of the network could become disconnected from the water source(s). Identification of such 'unintended isolation' requires a topological analysis of the network for each segment. A way to perform this topological analysis has been recently proposed by Giustolisi *et al*. (2008a). The key idea of the topological analysis is to solve the problem of finding nodes which are disconnected from water source(s) for networks such as those in Figure 3 or Figure 4. The analysis is restricted to valve subsets previously identified. Thus, the matrix $\bar{\mathbf{A}}_{pn\text{-}gap} = [\mathbf{B}_{pn} \mid \mathbf{V}_{pv}]$ must be constructed for each subset of valve-segments in order to check for other nodes and pipes ('unintended isolations') that could be disconnected from water source(s) due to already identified disconnected pipes in that segment; see Giustolisi *et al*. (2008a).

To perform this task, one needs to carry out a topological analysis using the hydraulic system equations similar to those used for network flow simulation. However, in this case, it does not matter what kind of expression is used for head loss because the aim is not to perform hydraulic simulation (i.e., to compute network state variables). Therefore, the hydraulic simulation model of Todini and Pilati (1988) was rewritten according to the following four assumptions (Giustolisi *et al*. 2008a,b): (1) a linear expression for the pipe hydraulic resistance; (2) matrix of pipe hydraulic resistance equal to one; (3) known nodal heads (source(s) of water) equal to one and; (4) nodal demands equal to zero.

These assumptions render the mathematical system of the hydraulic simulation model linear in nodal heads and pipe flows, because the roughness matrix $\mathbf{I}_{pp}$ $[\mathbf{I}_{pp} = \mathbf{R}_{pp}(k,k) = 1]$ is now independent of pipe flows. Consequently, by solving the following system of equations using the Moore–Penrose method (Golub and Van Loan 1993, Giustolisi *et al*. 2008a,b)

$$\begin{bmatrix} \mathbf{I}_{pp} & \mathbf{A}_{pn-gap} \\ \mathbf{A}_{np-gap} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{Q} \\ \mathbf{H} \end{bmatrix} = \begin{bmatrix} -\mathbf{A}_{p0-gap}\mathbf{I}_0 \\ \mathbf{0} \end{bmatrix} \Leftrightarrow \mathbf{AX} = \mathbf{B} \tag{5}$$

the set of disconnected nodes can be identified. The solution based on the Moore–Penrose pseudo-inverse method is one of the infinite number of possible solutions of an under-determinate linear system such as when there are one or more disconnected nodes (Figure 5). The method, as explained in Giustolisi *et al.* (2008a), assigns a zero value to all heads at disconnected nodes and a value of one to those that remain connected. Once the disconnected system nodes are identified, the matrices of the four building blocks of the simulation model can be automatically changed to detect disconnected pipes.

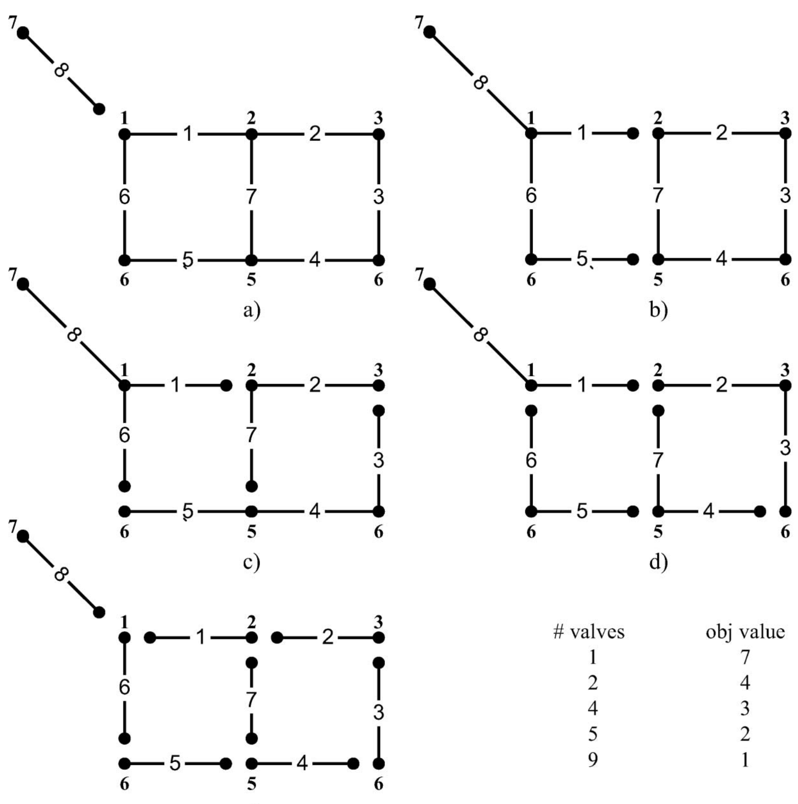
Thus, the topological analysis, further improved by Giustolisi *et al.* (2008b) in order to be applicable to large networks, requires the solution of the following equation:

$$\mathbf{H} = -\left(\mathbf{A}_{np-gap}\,\mathbf{A}_{pn-gap}\right)^{-1}\mathbf{A}_{np-gap}\,\mathbf{A}_{p0-gap}\,\mathbf{I}_0 \qquad (6)$$

where $\mathbf{A}_{np\text{-}gap}$ $\mathbf{A}_{pn\text{-}gap}$ is related to the Laplacian or Kirchhoff matrix of the network. In fact, it can be obtained by removing the columns and rows related to water source(s) node(s) from the Laplacian matrix.

The solution assigns a zero value to all the heads at disconnected nodes and a value of one to those that remain connected (Giustolisi *et al.* 2008a,b). Equation (6) is obtained by applying the above four assumptions to the Todini and Pilati (1988) hydraulic solution.

Finally, to further speed up the computational procedure, a solution of the linear system of Equation (6) was found by using the Cholesky incomplete factorisation for sparse matrices. This was then used as a pre-conditioner for a preconditioned conjugate gradients method (Saad 1996, Ch. 10) used to solve the system of linear equations (Equation 6). This system will be ill-conditioned when the matrix product $\mathbf{A}_{np\text{-}gap}$ $\mathbf{A}_{pn\text{-}gap}$ is not of full rank, i.e., when one or more nodes are disconnected from the water source(s). This property is related to the Laplacian matrix; the number of eigenvalues equal to zero represents the number of connected components (Brualdi and Ryser 1991). Therefore, a special form of Cholesky incomplete factorisation (named Cholesky-Infinity) must be used. This factorisation is based on the ordinary Cholesky factorisation which additionally handles real positive semi-definite matrices. When a zero pivot is encountered in the ordinary Cholesky factorisation, the diagonal of the Cholesky-Infinity factor is set to *Inf* (or the greatest real number of the computational environment) and the rest of that row is set to 0. This



# valves	obj value
1	7
2	4
4	3
5	2
9	1

Figure 5. Optimal isolation valve systems (including the number of valves and objective function values).

forces a zero in the corresponding entry of the solution vector in the associated system of linear equations (Zhang 1996).

Design of the isolation valve system

In order to prove the effectiveness of the algorithm, the optimal design of the isolation valve system of two networks has been performed. Optimisation was performed using the classic multi-objective genetic algorithm (Goldberg 1989). The decision variables were coded using a binary string (i.e., each solution as an individual) whose gene values indicate the presence or absence of valves $v(k,i)$ in the system. Thus, the number of binary variables equals $2n_p$ (to account for a possibility of a valve at each end of a pipe).

Case Study 1: the simple network of Figure 1

The optimal design for the example network in Figure 1 has been performed. This is done on a small system to allow verification of the results. The total number of variables is 16 in the example.

Two objectives were minimised during optimal design of the isolation valve system:

- The number of isolation valves in the network;
- The size of the largest segment (taken here as the segment with the largest number of pipes, i.e., assumes the same length for each pipe). Thus, this objective depends only on network topology (i.e., connectivity).

The use of the two straightforward objective functions allows easy analysis of optimisation results through the use of a very simple case study.

Furthermore, the optimisation was performed with and without considering unintended isolations. In the case of unintended isolations analysis, the trivial (for the analysis) case of isolation of pipe 8 has been excluded from calculating the second objective. This assumption is necessary in a network having a single source of water connected to the hydraulic system with one pipe. In fact, without that assumption, the second objective would always have the same maximum value corresponding to the disconnection of the entire network when pipe 8 is isolated. This assumption works as a constraint to isolate the tank pipe alone (pipe 8 in this case).

The exploration of the space of decision variables (size of $2^{16} = 65{,}536$) was performed using the maximum number of generations equal to 1,000 (20,000 evaluations of the objective function) in less than 1 minute (using a notebook computer with a Pentium Intel M 1.10 GHz processor) when unintended isolations where not considered. Clearly, when the objective function involves topological analysis related to each isolated segment the evaluation is slower and the same optimisation took about 2.5 minutes.

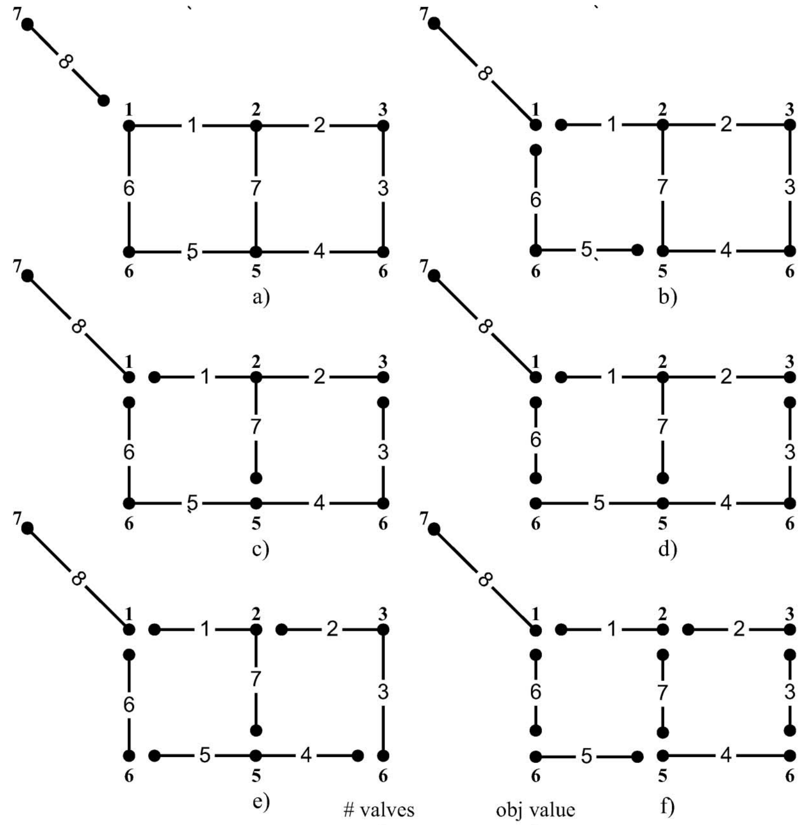
Finally, it is worth noting that a design with one source network does not represent good technical practice, but is used here to illustrate the methodology and test the capability of the optimisation algorithm.

Case Study 1: results and discussion

The results of the two optimisation problems (with or without considering unintended isolations) are reported graphically in Figures 5 and 6 where the objective function values (number of valves vs. maximum number of pipes) are also reported. Figure 5 shows that in order to minimise the maximum number of pipes among segment, in the case of one valve (see Figure 5a), it is necessary to locate the valve in the pipe 8 close to node 1. Using two valves located on pipes 1 and 5 results in four pipes in each of the two segments (see Figure 5b). The other (non-unique) configurations further reduce the maximum number of pipes among the segments using four and five valves (see Figure 5c and 5d). Finally, the use of at least nine valves is required to isolate each pipe separately (see Figure 5e). It is worth noting that this solution, the so called 'N-1 valve rule', is better than that obtained by following the 'N valve rule', as it suggests installation of one valve at each pipe connected to the each node except for one pipe (Ysusi 2000, Jun and Loganathan 2007).

Figure 6 shows the results of the optimal valve system design considering the source at node 7 and the topological analysis used to assess the actual portion of the network isolated from the tank for each isolated segment (unintended isolation concept). Note that the valve close to node 7 is not present in any of the optimal isolation valve solutions. This is caused by the form of the objective function that minimises the segment size and the number of valves. However, as valves are necessary for shutting large water sources they are not part of the optimal design procedure.

The first configuration (see Figure 6a), with one valve located on pipe 8 close to node 1, is equal to Figure 6a, being the only way to use one valve only. Each of the other five configurations using 3, 4, 5, 6 and 9 valves (see Figure 6b–f) require two valves close to node 1. Those valves isolate pipe 8, while they are never needed to isolate one of the other segments at the same time. Furthermore, the optimal designs did not generate unintended isolations (see Figure 6 and second objective of the optimisation, which is always equal to the maximum number of pipes in the segments). Clearly, this is only possible in a completely



# valves	obj value
1	7
3	5
4	4
5	3
6	2
9	1

Figure 6. Optimal isolation valve systems considering water source in 7 (including the number of valves and objective function values).

looped system as in this case study (Figure 1). Finally, the configuration with 9 valves (see Figure 6f) is again an 'N-1 valve rule' system, but able to disconnect each pipe without unintended isolations. Equally, the 9-valve design in Figure 5e is sub-optimal with respect to unintended isolations because the need to isolate pipe 6 results in disconnection from the source of the entire network.

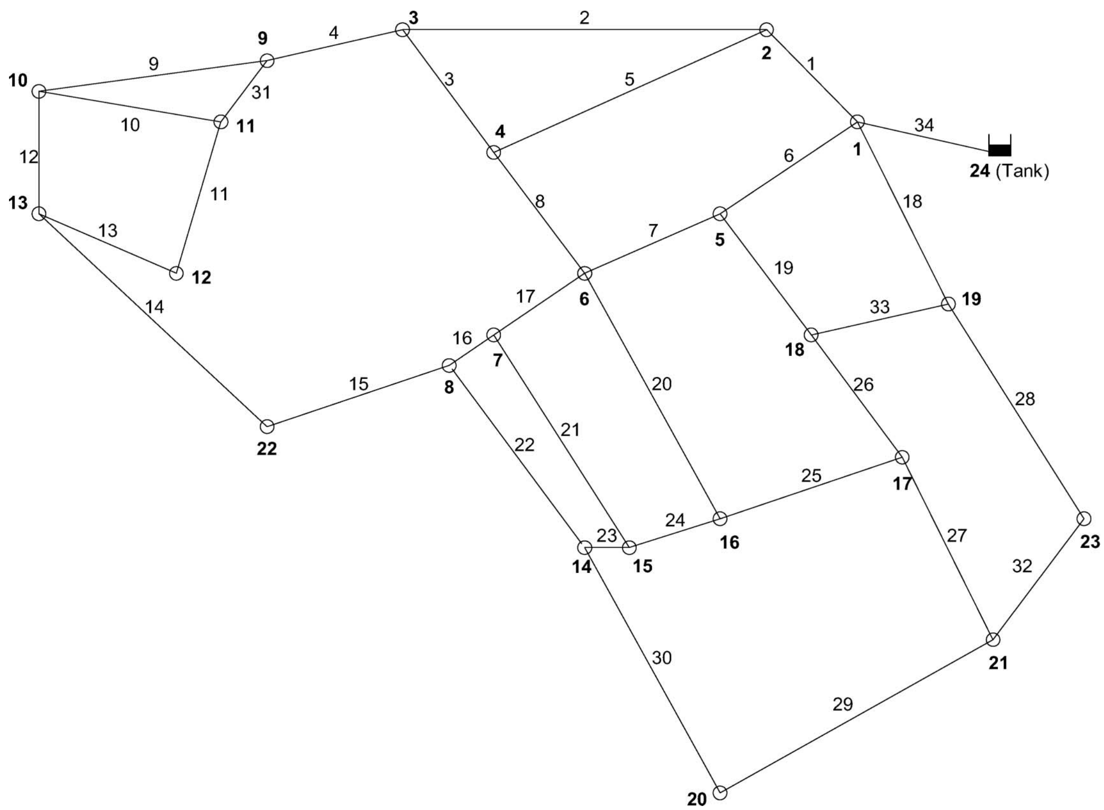
Case Study 2: Apulian network

The optimal design of the isolation valve system has been further performed for the Apulian network (Giustolisi *et al* 2008a) of Figure 7 using the same multi-objective genetic algorithm. Two objectives have been minimised: (i) the number of isolation valves and (ii) the maximum total undeliverable demand (for the entire network) considering unintended isolation (Jun and Loganathan 2007). The trivial case of pipe 34 isolation (causing disconnection of the entire network) has been excluded from the computation of the maximum undeliverable demand for the same reason previously reported for the simple network.

The number of isolation valves is considered here as an objective to influence the selection of segments based on network topology and demand distribution through pipes. Furthermore, the number of valves (in a given system and for assumed demand reliability) has direct impact on maintenance/management of the system. Clearly, the objective function here used could be substituted by the total cost of valve system depending on pipe diameters.

The complete data set for the network in Figure 7 can be found in Giustolisi *et al.* (2008a). Here only the pipe level demand of the network is reported in Figure 7. Furthermore, in order to reduce greatly the search space for the optimiser, a maximum of one or two valves for each pipe were tested. Consequently, two problems have been tested:

- Up to two valves can be allocated to each pipe. Thus, each gene is characterised by two states



Pipe No	Demand (l/s)
1	5.7
2	15.5
3	7.8
4	6.5
5	12.9
6	6.6
7	6.3
8	7.8
9	15.2
10	7.0
11	8.3
12	7.0
13	6.8
14	16.6
15	7.4
16	3.0
17	3.6
18	9.5
19	7.3
20	12.9
21	11.7
22	10.7
23	2.7
24	4.1
25	5.4
26	8.1
27	9.4
28	13.7
29	12.9
30	13.8
31	2.7
32	7.0
33	6.2
34	0.0

Figure 7. Apulian network layout.

(i.e., 0 or 1) and the chromosome length is equal to twice the number of pipes (= 68). The size of the search space is therefore equal to 2^{68} (= 2.9515×10^{20}). The optimisation run took about 24 minutes for 2000 GA generations using a notebook computer with a Pentium Intel M 1.10 GHz processor.

- Up to one valve can be allocated to each pipe. Thus, each gene is characterised by three states (valve not present, valve close to the node j of pipe k, valve close to the node i of the pipe k) and the chromosome length is equal to the number of pipes (=34). Consequently, the size of the search space is 3^{34} ($=1.6677 \times 10^{16}$). The optimisation run took about 10 minutes for 2000 GA generations using a notebook computer with a Pentium Intel M 1.10 GHz processor.

It is worth noting that the constraint imposed by using up to one valve for each pipe drastically reduces the search space (four orders of magnitude for this small-sized network) as well as the time required for the optimisation process. This is due to the shorter string length to be manipulated in the optimiser for the same number of generations.

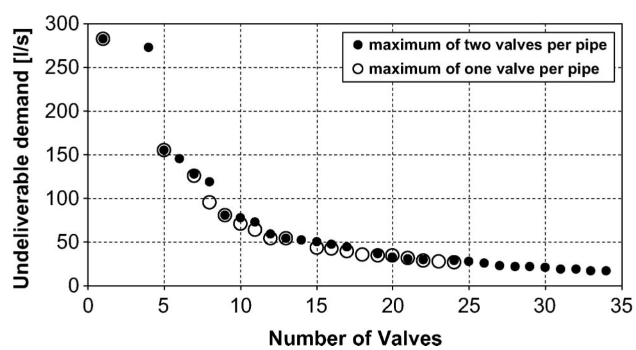


Figure 8. Optimal design of isolation valve system: Pareto front of solutions.

Case Study 2: results and discussion

Figure 8 shows the Pareto front of solutions found by the multi-objective GA using the two strategies, the maximum of one or two valves for each pipe. Each point of the graph is an optimal design (a set of locations for valves) showing the number of valves (i.e. a surrogate for cost) vs. the maximum total undeliverable demand due to intended (segment) and unintended isolations of the network.

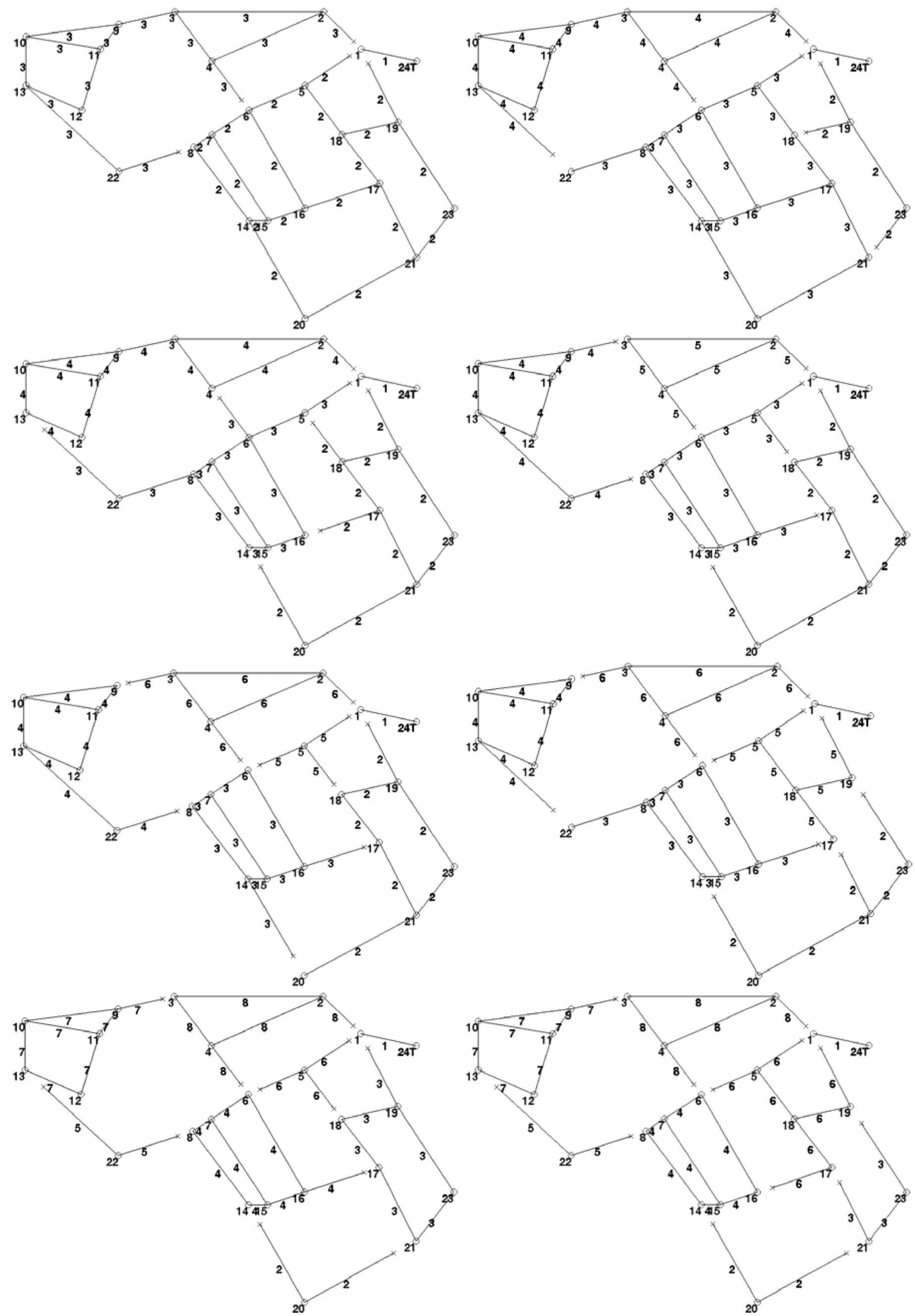


Figure 9. Optimal isolation valve system: [5, 7, 8, 9, 10, 11, 12, 13] valves.

Figure 8 demonstrates that the approach of using one valve as a constraint to reduce the search space for the optimal design is a good tactic when the number of valves is limited (24 valves in this case study as compared to 34 pipes). Clearly, the assumption of up to two valves for each pipe allows extending the Pareto front of solutions to those valve scenarios that have more than 24 valves. Thus this approach could reduce the undeliverable demand through the use of a larger number of valves. Therefore, the use of the approach which reduces the search space could be useful for medium- to large-sized networks and/or when the number of valves to be used is limited (i.e., less than the number of pipes), for example when the analysis aim is to design district metering areas or an isolation valve system for real-time control on threat diffusion.

To illustrate the case of only one valve for each pipe, Figure 8 shows that the solution with only one valve (located on pipe 34 close to node 1, $v(34,1)$) results in the network always being disconnected (pipe 34 excluded) when a pipe has to be isolated by means of the valve system (composed of one valve only in this case). Thus, the maximum undeliverable demand is the total demand required by the network (=282.1 l/s). On the other hand, by using 24 valves optimally located throughout the system, it is possible to reduce the maximum undeliverable demand to 26.7 l/s. This means that when one of the subsets of the valve system is used in order to isolate a segment, the undeliverable demand (unintended isolations included) is always less than 10% of the total demand. Furthermore, Figure 9 reports the optimal isolation valve systems for 8 valve sets [5, 7, 8, 9, 10, 11, 12, 13]. In the same pictures the number close to each pipe specifies its association to a segment number. For example, in the first network five valves generated three segments.

It is worth noting that all the solutions (the optimal isolation valve system) require the location of three valves close to node 1. Those valves are useful in order to isolate pipe 34 (the valve for the tank is not considered in the design), and also help to avoid the isolation of the entire network in other situations. It should be noted that the Apulian network has only one source of water and being able to isolate pipe 34 represents a trivial situation where all nodes are disconnected from the source. Therefore, it was not considered when computing the maximum undeliverable demand.

Once the multi-objective GA run has been performed, the selection of one preferred solution from the Pareto front (Figure 8) could be made based on the evaluation of network configurations (Figure 9) and the maximum undeliverable demand for one segment isolation. For example, if the isolation valve system composed of 13 elements was preferred, the maximum undeliverable demand is equal to 53.5 l/s and the network is composed of eight segments. This corresponds to a measure of mechanical reliability of the network, meaning that the isolation of one segment (due to planned or unplanned system maintenance) causes a maximum unsupplied demand of 53.5 l/s.

Recently, Giustolisi *et al.* (2008c) proved that the mechanical reliability in delivering water is dependent on both the actual isolation valve system and pipe sizing. Therefore, the pipe sizing exercise could be conducted considering the actual isolation valve system in order to maximise network reliability. Thus, a better approach to pipe sizing could use cost minimisation as the objective, subject to the constraint on minimum pressures for each network configuration generated by the actual isolation valve system scenario (isolation of eight segments in this case). In this way the maximum unsupplied demand (53.5 l/s in the case study considered here) due to disconnected pipes is a measure of the system reliability as it is the only reason for the unsupplied demand (Giustolisi *et al.* 2008c).

Conclusions

An algorithm for identifying the association between a subset of isolation valves and directly isolated segments has been presented. The algorithm is based on a modification of the original network topology accounting for existing isolation valves in the system. The algorithm has been illustrated using a small example network which has further served to demonstrate the multi-objective optimal design of valve systems. In addition, the algorithm has been proved to perform well by undertaking optimal design of an isolation valve system of a real system (the Apulian network). The two objectives used are the minimisation of the number of valves and the maximisation of the undeliverable demand. The authors envisage that the algorithm could be useful when considering reliability assessment for network design purposes, as for example in pipe sizing. Furthermore, the valve system analysis could be useful for contamination control in real time and for district metering area design for leakage control, for example.

Notations

The following symbols are used in the paper:

$\mathbf{A}$	=	coefficient matrix of the linear system for connectivity analysis
$\bar{\mathbf{A}}_{pn}$	=	general topological incidence matrix
$\bar{\mathbf{A}}_{pn\text{-}mod}$	=	modified general topological incidence matrix

$\bar{\mathbf{A}}_{pn\text{-}gap}$ = general topological incidence matrix of the network with gaps
$\mathbf{A}_{pn}$,$\mathbf{A}_{np}$,$\mathbf{A}_{p0}$ = topological incidence sub-matrices
$\mathbf{A}_{pn\text{-}gap}$,$\mathbf{A}_{np\text{-}gap}$,$\mathbf{A}_{p0\text{-}gap}$ = topological incidence sub-matrices of the network with gaps
$\mathbf{B}$ = known terms of the linear system for connectivity analysis
$\mathbf{B}_{pn}$,$\mathbf{V}_{pv}$,$\mathbf{V}_{vn}$,$\mathbf{V}_{vv}$ = building blocks of modified topological incidence matrix
$\mathbf{H}$ = vector of total network heads
$\mathbf{H}_0$ = vector of total fixed (i.e. known) network heads
$\mathbf{I}_{pp}$ or $\mathbf{I}$ = identity matrix
$\mathbf{I}_0$ = vector of ones water source(s) node(s)
$\mathbf{L}$ = line or edge graph matrix
i = index for nodes
k = index for pipes
m = exponent of $\mathbf{V}^m$, the 'transitive closure of the graph'
n_0 = total number of known heads
n_p = total number of network pipes
n_n = total number of network nodes
Ni = label of i-th node (internal or not)
Pk = label of k-th pipe
$\mathbf{Q}$ = vector of pipe flows
$\mathbf{R}_{pp}$ = diagonal matrix whose elements are pipe hydraulic resistance
$v(k,i)$ = elements of matrix $\mathbf{V}_{pn}$, valve on k-th pipe close to i-th node
$\mathbf{V}_{pn}$ = general topological matrix of the isolation valve system
$\mathbf{V}$ = adjacency matrix of pipes
$\mathbf{X}$ = unknown terms of the linear system for connectivity analysis
$\mathbf{0}$ = zero vector or matrix

Operators:

$()^{\mathrm{T}}$ = transpose operator
$()^{-1}$ = inversion operator

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